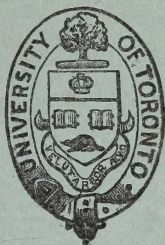


University of Toronto
FACULTY OF APPLIED SCIENCE AND ENGINEERING
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BULLETIN NO. 7

1927

SECTION No. 6

A Method of Base Line Measurement

By

L. B. Stewart

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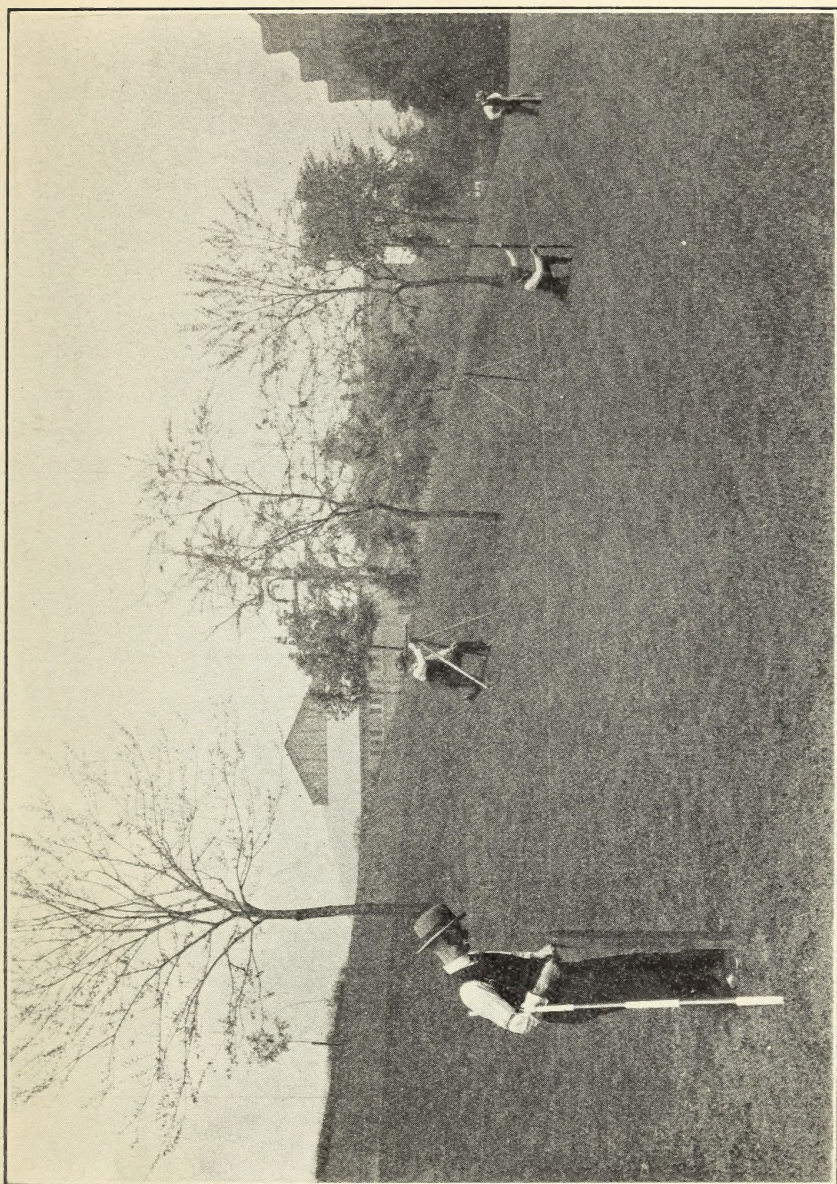
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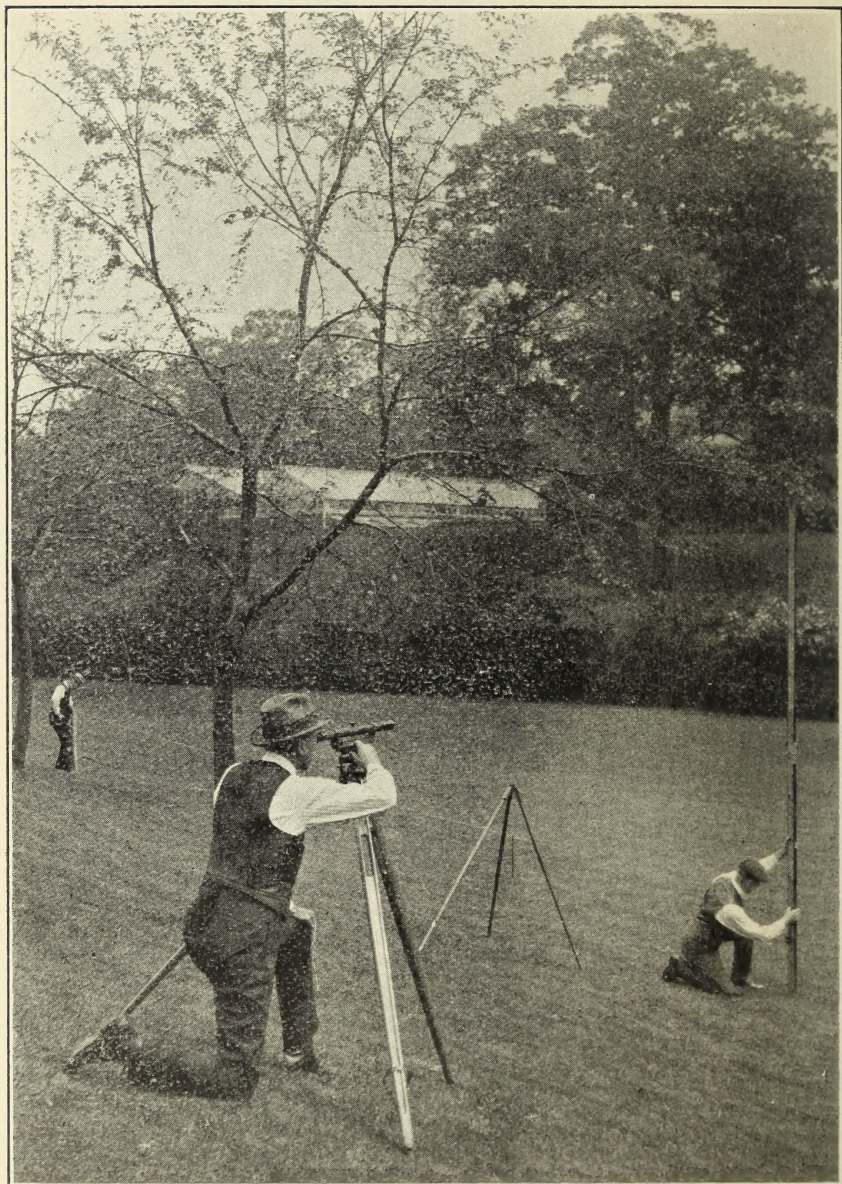
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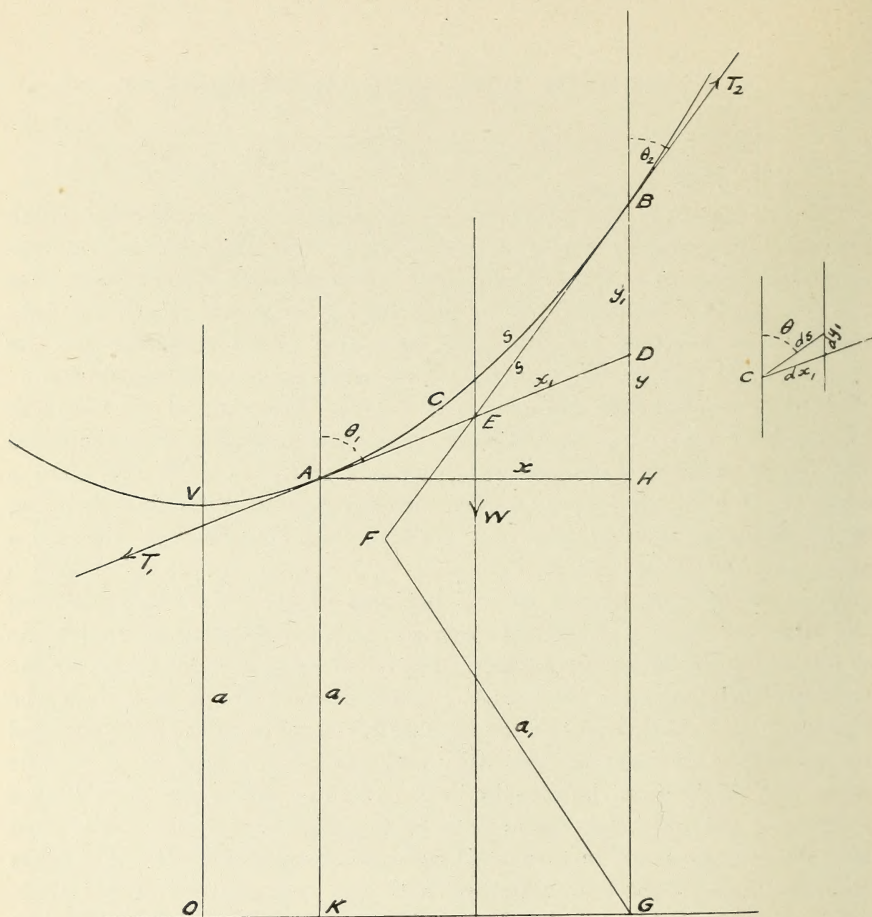


A METHOD OF BASE LINE MEASUREMENT

By L. B. STEWART

1. The object of this investigation is to develop a method by which precise linear measurements may be made with the ordinary instrumental equipment of the engineer. At the same time it is thought that the precision of the method to be described is such that it will not only meet all professional requirements, but also prove sufficient for the measurement of the base lines of a first order geodetic triangulation.

The method of measurement is as follows: The route of the base line having been cleared of obstructions, it is then marked out in the usual way by a series of carefully aligned posts spaced a trifle less than a tape-length apart and firmly driven into the ground, their tops standing 30 or 36 inches above the ground. To the top of each post is nailed a small piece of sheet zinc or copper, on which a fine line is ruled with a steel point at right angles to the direction of the line. In measuring the base the tape is stretched between each consecutive pair of posts in turn and the necessary tension applied to bring the zero lines of the tape precisely over the lines on the metal plates. While held thus, the height of the middle point of the tape with reference to the ends is found by holding a leveling rod vertically and against the tape at its middle point and reading on the rod the height of the tape, at the same time a reading of the rod being taken with an engineer's level; the rod is then held on the two posts in turn and readings taken with the level. These four readings of the rod, together with the temperature of the tape, constitute the field record. From these readings the heights of the higher end of the tape and its middle point above the lower end are found; and, in terms of these two heights, and the length of the tape, the extension of the tape due to the tension to which it has been subjected may be determined, and also the length of its projection on a horizontal plane. The constants of the tape: the weight of a unit of length, the stretch of a unit of length due to a unit tension, and the coefficient of expansion, must of course have been carefully determined. The derivation of the formulae by means of which the reductions above outlined may be made now follows; and in deriving them two assumptions are made that are not completely realized in practice, *viz.*, that the tape is perfectly flexible, and that the weight per unit of length is constant throughout notwithstanding the varying tension from point to point. The error due to the non-fulfilment of these conditions, however, must be extremely small.



2. In the figure AB represents a tape, stretched as in making a measurement. It is in equilibrium under the action of the tensions T_1 and T_2 , applied at its extremities, and its weight W acting vertically through its centre of gravity. The directions of these three forces pass through some point E . Then, taking A as the origin of coordinates, let x_1 and y_1 denote the oblique coordinates AD and DB of the point B , x and y the rectangular coordinates AH and HB , θ_1 and θ_2 the angles which the tangents to the curve at A and B make with the vertical, and s the length of the tape.

The sides of the triangle DEB are then parallel to, and, taken clockwise, in the same sense as, the directions of the three forces T_1 , T_2 and W , so that their lengths are proportional to those forces, and we have at once the equations:

$$T_1 = W \frac{\sin \theta_2}{\sin (\theta_1 - \theta_2)} \quad , \quad (1)$$

$$T_2 = W \frac{\sin \theta_1}{\sin (\theta_1 - \theta_2)} \quad , \quad (2)$$

whence $T_1 \sin \theta_1 = T_2 \sin \theta_2$. (3)

From (3) it follows that the horizontal component of the tension at any point of the tape is constant.

From (1) and (2) we have:

$$\begin{aligned} T_2 - T_1 &= W \frac{\sin \theta_1 - \sin \theta_2}{\sin (\theta_1 - \theta_2)} \quad , \\ &= ws \frac{\sin \theta_1 - \sin \theta_2}{\sin (\theta_1 - \theta_2)} \quad , \end{aligned} \quad (4)$$

in which w denotes the weight of a unit of length of the tape. Applying (4) to an element of the curve at any point C we have:

$$\begin{aligned} dT &= wds \frac{\sin \theta - \sin (\theta - d\theta)}{\sin d\theta} \quad , \\ &= wds \frac{d \sin \theta}{d\theta} = wds \cos \theta, \\ &= wdy; \end{aligned}$$

therefore, integrating between the limits y and 0 , we have

$$T_2 - T_1 = wy.$$

Comparing this result with (4) we have:

$$y = s \frac{\sin \theta_1 - \sin \theta_2}{\sin (\theta_1 - \theta_2)} \quad . \quad (5)$$

Also, denoting by a_1 the length of tape whose weight is equal to the tension T_1 we have

$$T_2 - wa_1 = wy,$$

or $T_2 = w(a_1 + y)$; (6)

from which we infer that the tension at any point of the curve is proportional to the vertical distance of the point from a horizontal line KG , where $AK = a_1$.

Again, for the element of the curve at C , we have

$$\frac{dx_1}{ds} = \frac{\sin \theta}{\sin \theta_1} \quad , \quad \frac{dy_1}{ds} = \frac{\sin (\theta_1 - \theta)}{\sin \theta_1}$$

Also, from (1), s' denoting the length AC ,

$$\begin{aligned} s' &= a_1 \frac{\sin (\theta_1 - \theta)}{\sin \theta} \\ &= a_1 (\sin \theta_1 \cot \theta - \cos \theta_1); \end{aligned}$$

whence, by differentiation,

$$\frac{ds}{d\theta} = -a_1 \frac{\sin \theta_1}{\sin^2 \theta}.$$

$$\therefore \frac{dx_1}{d\theta} = \frac{dx_1}{ds} \cdot \frac{ds}{d\theta} = -\frac{\sin \theta}{\sin \theta_1} \cdot \frac{a_1 \sin \theta_1}{\sin^2 \theta} = -\frac{a_1}{\sin \theta};$$

$$\begin{aligned} \frac{dy_1}{d\theta} &= \frac{dy_1}{ds} \cdot \frac{ds}{d\theta} = -\frac{\sin (\theta_1 - \theta)}{\sin \theta_1} \cdot \frac{a_1 \sin \theta_1}{\sin^2 \theta} = -a_1 \frac{\sin (\theta_1 - \theta)}{\sin^2 \theta} \\ &= -a_1 \sin \theta_1 \frac{\cos \theta}{\sin^2 \theta} + a_1 \frac{\cos \theta_1}{\sin \theta}. \end{aligned}$$

We therefore have,

$$\begin{aligned} x_1 &= a_1 \int_{\theta_2}^{\theta_1} \frac{d\theta}{\sin \theta} = a_1 (\log \tan \tfrac{1}{2}\theta_1 - \log \tan \tfrac{1}{2}\theta_2) \\ &= a_1 \log \frac{\tan \tfrac{1}{2}\theta_1}{\tan \tfrac{1}{2}\theta_2} \end{aligned} \quad (7)$$

$$\begin{aligned} \text{Also, } y_1 &= a_1 \int_{\theta_2}^{\theta_1} \left(\sin \theta_1 \frac{\cos \theta}{\sin^2 \theta} - \frac{\cos \theta_1}{\sin \theta} \right) d\theta \\ &= a_1 \sin \theta_1 \left(\frac{1}{\sin \theta_2} - \frac{1}{\sin \theta_1} \right) - a_1 \cos \theta_1 \log \frac{\tan \tfrac{1}{2}\theta_1}{\tan \tfrac{1}{2}\theta_2} \\ &= a_1 \frac{\sin \theta_1 - \sin \theta_2}{\sin \theta_2} - a_1 \cos \theta_1 \log \frac{\tan \tfrac{1}{2}\theta_1}{\tan \tfrac{1}{2}\theta_2} \end{aligned} \quad (8)$$

$$= a_1 \frac{\sin \theta_1 - \sin \theta_2}{\sin \theta_2} - x_1 \cos \theta_1 \quad (9)$$

Then, as $DH = x_1 \cos \theta_1$, we have

$$y = a_1 \frac{\sin \theta_1 - \sin \theta_2}{\sin \theta_2} \quad (10)$$

This last equation may also be obtained by writing (1) in the form,

$$a_1 = s \frac{\sin \theta_2}{\sin (\theta_1 - \theta_2)}, \quad (11)$$

and substituting the value of s from this in (5).

If the point A be at the vertex V of the curve, so that $\theta_1 = 90^\circ$, then the equations (11), (7), (10), (5), and (6), become, respectively (a denoting the length $V0$):

$$s = a \cot \theta_2 \quad (12)$$

$$x = a \log \cot \tfrac{1}{2}\theta_2 \quad (13)$$

$$y = a \frac{1 - \sin \theta_2}{\sin \theta_2} = a (\operatorname{cosec} \theta_2 - 1) \quad (14)$$

$$y = s \frac{1 - \sin \theta_2}{\cos \theta_2} = s (\sec \theta_2 - \tan \theta_2) \quad (15)$$

$$T_2 = w(a + y). \quad (16)$$

If a line GF now be drawn making an angle $\theta_1 - \theta_2$ with GH and intersecting the tangent at B in the point F , we have, in the triangle thus formed,

$$BF = (y + a_1) \frac{\sin (\theta_1 - \theta_2)}{\sin \theta_1},$$

$$GF = (y + a_1) \frac{\sin \theta_2}{\sin \theta_1}.$$

But (2) and (3) may be written:

$$s = (y + a_1) \frac{\sin (\theta_1 - \theta_2)}{\sin \theta_1},$$

$$a_1 = (y + a_1) \frac{\sin \theta_2}{\sin \theta_1};$$

hence it follows that $BF = s$, and $GF = a_1$. We therefore have the equation:

$$(y + a_1)^2 = s^2 + a_1^2 + 2sa_1 \cos \theta_1;$$

whence,

$$a_1 = \frac{s^2 + y^2}{2(y - s \cos \theta_1)} \quad (17)$$

3. Again, from (5),

$$\begin{aligned} \frac{s}{y} &= \frac{\sin (\theta_1 - \theta_2)}{\sin \theta_1 - \sin \theta_2}, \\ &= \frac{2 \sin \frac{1}{2}(\theta_1 - \theta_2) \cos \frac{1}{2}(\theta_1 - \theta_2)}{2 \sin \frac{1}{2}(\theta_1 - \theta_2) \cos \frac{1}{2}(\theta_1 + \theta_2)} \\ &= \frac{1 + \tan \frac{1}{2}\theta_1 \tan \frac{1}{2}\theta_2}{1 - \tan \frac{1}{2}\theta_1 \tan \frac{1}{2}\theta_2}; \end{aligned}$$

whence
$$\frac{s+y}{s-y} = \frac{1}{\tan \frac{1}{2}\theta_1 \tan \frac{1}{2}\theta_2}$$

or
$$\tan \frac{1}{2}\theta_1 \tan \frac{1}{2}\theta_2 = \frac{s-y}{s+y}. \quad (18)$$

Again, let y' denote the ordinate to the curve at its middle point; then by (17),

$$2a_1 = \frac{s^2 - y^2}{y - s \cos \theta_1} = \frac{s^2 - 4y'^2}{4y' - 2s \cos \theta_1},$$

so that, solving for θ_1 , we find

$$\cos \theta_1 = \frac{(s^2 - y^2) 4y' - (s^2 - 4y'^2)y}{(s^2 - y^2)2s - (s^2 - 4y'^2)s} \quad (19)$$

Again, writing

$$m = \frac{\tan \frac{1}{2}\theta_1}{\tan \frac{1}{2}\theta_2}, \quad (20)$$

and expanding $\log m$, (7) becomes

$$\frac{x_1}{a_1} = 2 \left\{ \frac{m-1}{m+1} + \frac{1}{3} \left(\frac{m-1}{m+1} \right)^3 + \frac{1}{5} \left(\frac{m-1}{m+1} \right)^5 + \right\} \quad (21)$$

Also

$$\frac{m-1}{m+1} = \frac{\tan \frac{1}{2}\theta_1 - \tan \frac{1}{2}\theta_2}{\tan \frac{1}{2}\theta_1 + \tan \frac{1}{2}\theta_2};$$

so that, eliminating θ_2 by (18), and making use of the equation:

$$\tan^2 \frac{1}{2}\theta_1 = \frac{1 - \cos \theta_1}{1 + \cos \theta_1},$$

we find, after a little reduction,

$$\frac{m-1}{m+1} = \frac{y-s \cos \theta_1}{s-y \cos \theta_1}. \quad (22)$$

Also, introducing the value of $\cos \theta_1$ in (19) and reducing, we have

$$\frac{m-1}{m+1} = \frac{2s(y-2y')}{s^2-4y'(y-y')}. \quad (23)$$

Substituting now in (21),

$$x_1 = \frac{x}{\sin \theta_1},$$

and the value of a_1 in (17), we obtain:

$$x = \frac{(s^2-y^2) \sin \theta_1}{s-y \cos \theta_1} \left\{ 1 + \frac{1}{3} \left(\frac{m-1}{m+1} \right)^2 + \frac{1}{5} \left(\frac{m-1}{m+1} \right)^4 + \right\} \quad (24)$$

Then, for brevity, writing

$$M = \frac{m-1}{m+1},$$

equation (22) gives

$$\cos \theta_1 = \frac{y-Ms}{s-My},$$

which, substituted in (24), give on reduction,

$$x = \sqrt{s^2-y^2} \cdot (1-M^2)^{\frac{1}{2}} \cdot (1 + \frac{1}{3}M^2 + \frac{1}{5}M^4 +).$$

Then, expanding, multiplying out and rearranging, we have finally:

$$x = \sqrt{s^2-y^2} \left(1 - \frac{1}{6}M^2 - \frac{11}{120}M^4 - \frac{103}{1680}M^6 - \frac{1823}{40320}M^8 - \right). \quad (25)$$

We shall now apply this expression to a numerical example in order to determine the relative magnitudes of its terms, taking as data:

$$s = 100, y = 50, y' = 24;$$

we then find—

$$\begin{aligned} \text{1st term} &= 86.60254 \\ -2\text{nd } &'' = -0.0410123\dots \\ -3\text{rd } &'' = -0.0000541\dots \\ -4\text{th } &'' = -0.0000001218\dots \\ -5\text{th } &'' = -0.000000000255\dots \end{aligned}$$

$$x = 86.56146\dots$$

∴

By means of equation (42) we find, by assuming $w = 0.012$ lb., that the tension in this case is

$$11.556 \text{ pounds.}$$

If the tension be 20 pounds, s and y remaining unchanged, equation (45) gives

$$y' = 24.446\dots;$$

so that, taking as data:

$$s = 100, y = 50, y' = 24.45,$$

we find the following values for the terms of (25):

$$\begin{aligned} \text{1st term} &= 86.60254\dots \\ -2\text{nd } &'' = -0.01241544\dots \\ -3\text{rd } &'' = -0.0000058736\dots \\ -4\text{th } &'' = -0.000000003379\dots \\ -5\text{th } &'' = -0.000000000021435\dots \end{aligned}$$

∴

$$x = 86.59012\dots$$

In this case it appears that the third term amounts to less than 6 parts in 100000000, the tape length being the unit; so that for practical use we are safe in writing:

$$x = \sqrt{(s+y)(s-y)} \cdot \left(1 - \frac{1}{6}M^2\right), \quad (26)$$

where,

$$M = \frac{2s(y-2y')}{s^2 - 4y'(y-y')}.$$

As an additional example we shall take the following:

$$s = 100, y = 0, y' = -0.7,$$

so that the ends of the tape are at the same height. In this case the equations become:

$$x = s \left(1 - \frac{1}{6}M^2 - \frac{11}{120}M^4 - \dots \right) \quad (27)$$

$$M = -\frac{4y's}{s^2 + 4y'^2}$$

and the result is:

$$\begin{array}{rcl}
 \text{1st term} & = & 100 \\
 -2\text{nd} \quad " & = & -0.013061538 \dots \\
 -3\text{rd} \quad " & = & -0.00000562993 \dots \\
 \hline
 & & 99.986932832 \dots \\
 \hline
 \end{array}$$

We may check this result by computing the same example by means of the well-known expression:

$$s - x = \frac{8}{3} \frac{y'^2}{s} + \frac{32}{15} \frac{y'^4}{s^3},$$

by which we find:

$$x = 99.986932825 \dots,$$

the omitted terms in the two expansions only affecting the eighth decimal place.

4. It may be well at this point to compare the precision to be attained by the above method of determining the projection of the tape length on a horizontal plane, with that of the usual method in which the tension applied to the tape is measured by means of a spring balance or weights, and x is found in terms of the tension and the weight and length of the tape. This may be effected by finding the error in x due to an error in the observed quantity in each method. Thus, from (27) we have,

$$\begin{aligned}
 \frac{dx}{dy'} &= \frac{dx}{dM} \frac{dM}{dy'} = -\frac{1}{3} s M \frac{4s(s^2 - 4y'^2)}{(s^2 + 4y'^2)^2} \\
 &= -\frac{16}{3} \frac{y' s^3}{(s^2 + 4y'^2)^3}.
 \end{aligned}$$

Then, applying this to the last example: $s = 100$, $y = 0$, $y' = -0.7$, we find:

$$\frac{dx}{dy'} = 0.037304 \dots;$$

so that, if $dy' = 0.005$, $dx = 0.0001865 \dots$

In order to compare this result with that of an error in reading a spring balance we take the well-known equations:

$$\begin{aligned}
 s - x &= \frac{s}{24} \left(\frac{W}{T} \right)^2 \\
 s - x &= \frac{8}{3} \frac{y'^2}{s}
 \end{aligned} \tag{28}$$

and equating them, we find,

$$T = \frac{Ws}{8y'}.$$

Also,

$$\frac{dT}{dx} = \frac{sW^2}{48T(s-x)^2} = \frac{12T^2}{sW^2}.$$

Then, assuming $W=1.2$ pounds, and $dx=0.0001865$, we find:

$$T=21.42857 \text{ pounds,}$$

$$dT=0.15294 \text{ pounds.}$$

It appears then that, with the assumed data, an error of 0.005 ft. in measuring y' will cause the same error in x as an error of 0.153 pound in reading a spring balance.

5. We proceed next to determine the extension of the tape due to the tension to which it is subjected in making a measurement. Returning to equation (6) and denoting by

s_0 the total length AB of the tape,

y the ordinate of any point C on the tape,

s the length AC , and

T the tension at the point C ,

we have

$$T=ew(y+a_1).$$

Then if the same construction be made about the point C as about B , viz., a tangent to the curve be drawn at C having the length s , and also a vertical line terminating in OG , and the point of intersection with OG be joined to the extremity of the tangent, the triangle thus formed will give the relation:

$$(a_1+y)^2=s^2+a_1^2+2sa_1 \cos \theta_1.$$

If also,

e denote the extension of a unit of length of the tape due to unit tension, and

E the total extension of the tape,

we have,

$$\begin{aligned} eTds &= ew(a_1+y)ds \\ &= ew\sqrt{(s^2+2sa_1 \cos \theta_1+a_1^2)}ds; \end{aligned}$$

also

$$E=ew \int_0^{s_0} (s^2+2sa_1 \cos \theta_1+a_1^2)ds.$$

To integrate this we substitute,

$$s+a_1 \cos \theta_1 = z,$$

so that,

$$ds=dz$$

and

$$\begin{aligned} s^2+2sa_1 \cos \theta_1+a_1^2 \cos^2 \theta_1 &= z^2 \\ s^2+2sa_1 \cos \theta_1+a_1^2 &= z^2-a_1^2 \cos^2 \theta_1+a_1^2 \\ &= z^2+a_1^2 \sin^2 \theta_1 \\ &= z^2+b^2 \text{ (assume).} \end{aligned}$$

Then as,

$$\int (z^2+b^2)dz = \frac{z\sqrt{(z^2+b^2)}}{2} + \frac{b^2}{2} \log \left\{ z + \sqrt{(z^2+b^2)} \right\},$$

we have, on introducing the limits and omitting the subscript of s_0 ,

$$E = \frac{ew}{2} \left\{ (s + a_1 \cos \theta_1) \sqrt{(s^2 + 2sa_1 \cos \theta_1 + a_1^2) - a_1^2 \cos \theta_1} \right. \\ \left. + a_1^2 \sin^2 \theta_1 \log \frac{s + a_1 \cos \theta_1 + \sqrt{(s^2 + 2sa_1 \cos \theta_1 + a_1^2)}}{a_1 \cos \theta_1 + a_1} \right\}. \quad (29)$$

This may be simplified by substituting,

$$\cos \theta_1 = \frac{y - Ms}{s - My},$$

and the value of a_1 given by (17), which becomes,

$$a_1 = \frac{s - My}{2M};$$

so that we have after reduction,

$$E = \frac{ew}{2} \left\{ \frac{s^2 + y^2}{2M} + \frac{(s^2 - y^2)(1 - M^2)}{4M^2} \log m \right\} \\ = \frac{ew}{2} \left\{ \frac{s^2 + y^2}{2M} + (s^2 - y^2) \frac{1 - M^2}{4M^2} \cdot 2(M + \frac{1}{3}M^3 + \frac{1}{5}M^5 + \dots) \right\} \\ = \frac{ew}{2} \left\{ \frac{s^2}{M} - (s + y)(s - y) \left(\frac{1}{3}M + \frac{1}{15}M^3 + \dots \right) \right\} \quad (30)$$

Applying now equation (30) to the following three examples:

$$s = 100, y = 50, y' = 24.3;$$

$$s = 100, y = 10, y' = 4.3;$$

$$s = 100, y = 0, y' = -0.7;$$

and assuming, $w = 0.012$, $e = 0.00000911$, we find the following values for E , respectively:

$$0.0146398 \dots,$$

$$0.0193249 \dots,$$

$$0.0195202 \dots;$$

the values of the second term of (30) being, respectively:

$$0.0000051003 \dots$$

$$0.0000051003 \dots$$

$$0.0000051003 \dots$$

As each of these values amounts to only about 5 parts in 100000000, that term and the following may safely be neglected, and equation (30) written in the simple form:

$$E = \frac{ew}{2} \frac{s^2}{M}. \quad (31)$$

6. The corrections for change of length of the tape due to tension and temperature may be applied directly to s before computing x ; a simpler method, however, is to compute x , using the standard length of the tape, and then apply a correction to the value thus found, due to

tension and temperature. The relation between small changes in x and s may be found as follows: Taking log's. in (26) and differentiating, we have,

$$\log x = \frac{1}{2} \log (s^2 - y^2) + \log (1 - \frac{1}{6} M^2),$$

$$= \frac{1}{2} \log (s^2 - y^2) - \frac{1}{6} M^2 - ,$$

and

$$\frac{1}{x} \frac{dx}{ds} = \frac{1}{2} \frac{2s}{s^2 - y^2} - \frac{1}{3} M \frac{dM}{ds} ,$$

or

$$\frac{dx}{ds} = \frac{sx}{s^2 - y^2} - \frac{x}{3} M \frac{dM}{ds} .$$

Also

$$M = 2(y - 2y') \frac{s}{s^2 - 4y'(y - y')}$$

and

$$\frac{dM}{ds} = 2(y - 2y') \frac{s^2 - 4y'(y - y') - 2s^2}{\{s^2 - 4y'(y - y')\}^2}$$

$$= -\frac{M}{3} \cdot \frac{s^2 + 4y'(y - y')}{s^2 - 4y'(y - y')} .$$

Therefore,

$$\frac{dx}{ds} = \frac{sx}{s^2 - y^2} + \frac{x}{3} \frac{M^2}{s} \cdot \frac{s^2 + 4y'(y - y')}{s^2 - 4y'(y - y')} .$$

Then, substituting the value of x and omitting powers of M above the second, this becomes:

$$\frac{dx}{ds} = \frac{s}{\sqrt{s^2 - y^2}} (1 - \frac{1}{6} M^2) + \frac{M^2}{3} \frac{\sqrt{s^2 - y^2}}{s} \cdot \frac{s^2 + 4y'(y - y')}{s^2 - 4y'(y - y')} \quad (32)$$

If we now apply this equation to the above three examples, assuming $ds = 0.03$, the terms containing M are found to be as follows, respectively:

0.00001203...

0.00000410...

0.00000392...;

which shows that these terms are not likely to amount to one part in 10000000, and may therefore be neglected. We may therefore, without material error, write (32) in the simple form:

$$\frac{dx}{ds} = \frac{s}{\sqrt{(s+y)(s-y)}} \quad (33)$$

The precision of this method of correcting for increase of length of the tape, in which the change in s and the resulting change in x are treated as infinitesimals, may be tested numerically by applying the correction dx for the first example just considered, *viz.*,

$$dx = 0.0346531 \dots ,$$

to the value of x found above for the same data:

$$x' = 86.5824180 \dots$$

giving the corrected value:

$$x = 86.6170711 \dots ;$$

and then computing x directly from the data:

$$s = 100.03, y = 50, y' = 24.3;$$

whence we find

$$x = 86.6170694 \dots;$$

which agrees satisfactorily with the former value.

The correction for change of temperature is found by the expression:

$$as(t - t_0),$$

in which,

a denotes the coefficient of expansion of the tape,

t the temperature of the tape during a measurement, and

t_0 the temperature at which its length is standard when subjected

to no tension.

7. The complete equation for x may therefore be written:

$$x = \sqrt{(s+y)(s-y)} \cdot \left(1 - \frac{1}{6}M^2\right) + \left\{ \frac{ew}{2} \cdot \frac{s^2}{M} + as(t - t_0) \right\} \frac{s}{\sqrt{(s+y)(s-y)}}; \quad (34)$$

in which

$$M = \frac{2s(y - 2y')}{s^2 - 4y'(y - y')}.$$

A table is appended to this paper, giving the value of M for values of y up to 10 feet, the length of the tape being 100 feet, the arguments of the table being y and $\frac{1}{2}y - y'$, the latter quantity being the approximate value of the middle ordinate contained between the curve and its chord.

The following examples give a form for the field record and show the application of equation (34).

From stake to stake	Length of tape	Rod readings				Ther. F.
		Back stake	Forward stake	Middle of tape		
				Inst.	Tape	
27-28	100	6.134	1.271	7.427	3.141	72° .2
35-36	100	0.747	1.985	4.609	2.118	73° .1

To find y and $\frac{1}{2}y - y'$, let

A and B denote readings of rod on lower and higher stakes, respectively,

C the reading of rod when held at middle of tape, and

D the reading of the tape on the rod.

Then

$$y = A - B \quad (35)$$

$$y' = A - (C - D) = D + A - C$$

$$\frac{1}{2}y - y' = \frac{A - B}{2} + C - A - D \quad (36)$$

The constants of the tape may be assumed to be:

$$e = 0.00000911$$

$$w = 0.012$$

$$a = 0.0000065$$

$$t_0 = 62^\circ \text{ F.}$$

Example 1.

$$\begin{array}{rcl}
 y = & 4.863 & \frac{1}{2}y - y' = 0.583 \\
 s + y = & 104.863 & \\
 s - y = & 95.137 & \\
 \log (s + y) = & 2.0206223 & \log (\frac{1}{2}ews^2) = \overline{4}.73767 \\
 \text{" } (s - y) = & 1.9783495 & \text{" } M = \overline{2}.36870 \\
 & \hline
 & 3.9989718 & \text{" } 0.023387 = \overline{2}.36897 \\
 & \hline
 \text{" } 1\text{st } t = & 1.9994859 & \text{" } (as) = \overline{4}.81291 \\
 \text{" } \frac{1}{6} = & \overline{1}.22185 & \text{" } (t - t_0) = \overline{1}.00860 \\
 \text{" } M^2 = & \overline{4}.73739 & \text{" } 0.006629 = \overline{3}.82151 \\
 & \hline
 \text{" } 2\text{nd } t = & \overline{3}.95873 & \text{" } 0.030016 = \overline{2}.47735 \\
 & \hline
 1\text{st } t = & 99.88170 & \text{" } s = 2 \\
 2\text{nd } t = & 0.00909 & \text{" } \sqrt{s^2 - y^2} = 1.99949 \\
 & \hline
 & 99.87261 & \text{" } 0.03005 = \overline{2}.47786 \\
 \text{Corr.} = & 0.03005 & \\
 & \hline
 x = & 99.90266 & \\
 & \hline
 \end{array}$$

Example 2.

$$\begin{array}{rcl}
 y = & 0.238 & \frac{1}{2}y - y' = 0.625 \\
 s + y = & 100.238 & \\
 s - y = & 99.762 & \\
 \log (s + y) = & 2.0010324 & \log (\frac{1}{2}ews^2) = \overline{4}.73767 \\
 \text{" } (s - y) = & 1.9989651 & \text{" } M = \overline{2}.39787 \\
 & \hline
 & 3.9999975 & \text{" } 0.021867 = \overline{1}.33980 \\
 & \hline
 \text{" } 1\text{st } t = & 1.9999987 & \text{" } as = \overline{4}.81291 \\
 \text{" } \frac{1}{6} = & \overline{1}.22185 & \text{" } (t - t_0) = \overline{1}.04532 \\
 \text{" } M^2 = & \overline{4}.79574 & \text{" } 0.007215 = \overline{3}.85823 \\
 & \hline
 & & \\
 & \hline
 \end{array}$$

" 2nd t	$= \overline{2.01759}$	" 0.029082	$= \overline{2.46362}$
1st t	$= \overline{99.99970}$	" s	$= 2$
2nd t	$= 0.01041$	" $\sqrt{(s^2 - y^2)}$	$= 2$
	$\overline{99.98929}$	" 0.02908	$= \overline{2.46362}$
Corr.	$= 0.02908$		
x	$= \overline{100.01837}$		

8. The constants of the tape e and w may be obtained in the following manner: The tape is stretched as in making a measurement, with its ends resting on firm supports which are at the same elevation. While one end is held fast the other end is free to move as different tensions are applied to it in a horizontal direction. The tensions are best applied by means of weights attached by a cord to the end of the tape, the cord passing over a frictionless pulley. Two widely different tensions are then applied to the tape and the horizontal movement of the free end measured accurately, and also the ordinates y' and y'' of the middle point of the tape corresponding to the two tensions. We have then the following equations:

$$x_1 = s_1(1 - \frac{1}{6}M_1^2), \text{ where } M_1 = \frac{4y's}{s^2 - 4y'^2};$$

$$x_2 = s_2(1 - \frac{1}{6}M_2^2), \text{ where } M_2 = \frac{4y''s}{s^2 - 4y''^2}.$$

In computing M_1 and M_2 the standard length s of the tape may be used. These equations give:

$$s_1 = x_1 + \frac{1}{6}sM_1^2, \quad s_2 = x_2 + \frac{1}{6}sM_2^2;$$

$$\text{whence} \quad s_2 - s_1 = (x_2 - x_1) + \frac{1}{6}s(M_2^2 - M_1^2). \quad (37)$$

$$\text{Then,} \quad e = \frac{s_2 - s_1}{s(T_2 - T_1)}. \quad (38)$$

To find w , we have by transposing one of (23):

$$W^2 = \frac{24T^2(s - x)}{s}. \quad (39)$$

As an example, let us assume that tensions of 10 and 50 pounds are applied in turn to a tape, and it is found that,

$$x_2 - x_1 = 0.0939,$$

$$y' = 1.500,$$

$$y'' = 0.300.$$

We then find:

$$\begin{aligned}s_2 - s_1 &= 0.0939 \\ &+ 0.0023998 \\ &- 0.059892 \\ &\hline &= 0.036408 \\ &\hline\end{aligned}$$

$$\therefore e = \frac{0.0364}{100 \times 40} = 0.0000091.$$

In finding w we have the two values of $s - x$:

$$s_1 - x_1 = \frac{1}{6}sM_1^2, \quad s_2 - x_2 = \frac{1}{6}sM_2^2.$$

These give,

$$\begin{aligned}W &= 1.19892 \\ W &= 1.19974 \\ &\hline\end{aligned}$$

$$\text{Mean} = 1.19933$$

$$\therefore w = 0.01199$$

9. The method of reducing base line measurements developed above, and embodied in equation (34), may readily be applied to measurements in which the tension is found by means of a spring balance or weights, and the correction to reduce to the chord length of the tape determined in terms of that tension and the length and weight of the tape. To effect this it is merely necessary to express the quantity M in terms of s , y , w , and T — T denoting the tension at the forward end of the tape. Thus from (17),

$$\cos \theta_1 = \frac{y}{s} - \frac{s^2 - y^2}{2a_1s},$$

which, substituted in (22), gives on reduction,

$$M = \frac{s}{2a_1 + y}. \quad (40)$$

Also, by (6),

$$a_1 = \frac{T}{w} - y;$$

whence,

$$M = \frac{ws}{2T - wy} = \frac{W}{2T - wy} \quad (41)$$

Also

$$T = \frac{w}{2} \left(y + \frac{s}{M} \right) \quad (42)$$

Thus M may be found by (41) and used in (34) to determine x , the remainder of the computation being unchanged.

If the above value of M be substituted in (31) it becomes

$$E = es(T - \frac{1}{2}wy); \quad (43)$$

then as

$$wy = T_2 - T_1,$$

we have

$$E = es \frac{T_1 + T_2}{2} \quad (44)$$

from which it appears that,

$$\frac{1}{2}(T_1 + T_2) \text{ or } T - \frac{1}{2}wy$$

expresses, very closely, the mean value of the tension throughout the length of the tape.

If T is given, and also s , y , and w , we may find y' , if required, as follows: we have,

$$\cos \theta_1 = \frac{y}{s} - \frac{s^2 - y^2}{2a_1 s};$$

also by (17),

$$(a_1 + y')^2 = a_1^2 + \left(\frac{s}{2}\right)^2 + 2a_1 \frac{s}{2} \cos \theta_1;$$

or, substituting $\cos \theta_1$,

$$(a_1 + y')^2 = \left(a_1 + \frac{y}{2}\right)^2 - \frac{s^2 - y^2}{4} \quad (45)$$

a_1 being given by the equation:

$$a_1 = \frac{T}{w} - y.$$

10. The following are some of the advantages of the method of base line measurement above described:

Only four men are required for the measurement of a base;

No special training is necessary on the part of the two tape men, only good eye-sight;

No divided scales need be read at the ends of the tape, thus diminishing the chance of error;

The formulæ which have been derived are applicable to practically any slope.

With reference to this last point, however, it must be noted that the steepness of the slope on which this, or any, method may be used depends upon the precision of the leveling operations. Thus, by a process similar to that of p. 300 we find—omitting terms containing M :

$$\frac{dx}{dy} = -\frac{y}{\sqrt{s^2 - y^2}};$$

so that, making

$$\frac{dx}{dy} = 0.1, \text{ and } s = 100,$$

we find

$$y = 9.95;$$

which is the value of y for which the error in x is equal to one-tenth of the error in y .

The following table gives the value of M in terms of y and $\frac{1}{2}y - y'$, from which any required value may be interpolated, for values of y up to 10 feet, assuming $s = 100$ feet.

TABLE
 $\frac{1}{2}y-y'$

y	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	Diff.
0.0	0.012000	0.015999	0.019998	0.023997	0.027995	0.031992	0.035988	0.039984	0.043979	0.047972	3993-99
.1	0.012000	0.015999	0.019998	0.023997	0.027995	0.031992	0.035988	0.039984	0.043979	0.047972	3993-99
.2	0.012000	0.015999	0.019998	0.023997	0.027995	0.031992	0.035988	0.039984	0.043979	0.047973	3994-99
.3	0.012000	0.015999	0.019998	0.023997	0.027995	0.031992	0.035989	0.039984	0.043979	0.047973	3994-99
.4	0.012000	0.015999	0.019998	0.023997	0.027995	0.031992	0.035989	0.039985	0.043979	0.047973	3994-99
.5	0.012000	0.015999	0.019999	0.023997	0.027995	0.031993	0.035989	0.039985	0.043980	0.047974	3994-99
.6	0.012000	0.016000	0.019999	0.023997	0.027996	0.031993	0.035990	0.039985	0.043980	0.047974	3994-00
.7	0.012000	0.016000	0.019999	0.023998	0.027996	0.031993	0.035990	0.039986	0.043981	0.047975	3994-00
.8	0.012000	0.016000	0.019998	0.023998	0.027997	0.031994	0.035991	0.039987	0.043982	0.047975	3994-00
.9	0.012000	0.016000	0.019999	0.023998	0.027997	0.031994	0.035991	0.039987	0.043982	0.047976	3994-00
1.0	0.012001	0.016001	0.020000	0.023999	0.027997	0.031995	0.035992	0.039988	0.043983	0.047977	3994-00
.1	0.012001	0.016001	0.020000	0.023999	0.027998	0.031996	0.035993	0.039989	0.043984	0.047978	3994-00
.2	0.012001	0.016001	0.020001	0.024000	0.027999	0.031996	0.035994	0.039990	0.043985	0.047979	3994-00
.3	0.012002	0.016002	0.020001	0.024001	0.027999	0.031997	0.035994	0.039991	0.043986	0.047980	3994-00
.4	0.012002	0.016002	0.020002	0.024001	0.028000	0.031998	0.035995	0.039992	0.043987	0.047982	3995-00
.5	0.012002	0.016002	0.020003	0.024002	0.028001	0.031999	0.035996	0.039993	0.043989	0.047983	3994-00
.6	0.012003	0.016003	0.020003	0.024003	0.028002	0.032000	0.035999	0.039994	0.043990	0.047985	3995-00
.7	0.012003	0.016004	0.020004	0.024003	0.028003	0.032001	0.035999	0.039996	0.043991	0.047986	3995-01
.8	0.012003	0.016004	0.020004	0.024004	0.028004	0.032002	0.036000	0.039997	0.043993	0.047988	3995-01
.9	0.012004	0.016005	0.020005	0.024005	0.028005	0.032003	0.036001	0.039998	0.043995	0.047990	3995-01
2.0	0.012004	0.016005	0.020006	0.024006	0.028006	0.032005	0.036003	0.040000	0.043996	0.047992	3996-01
.1	0.012005	0.016006	0.020007	0.024007	0.028007	0.032006	0.036004	0.040002	0.043998	0.047994	3996-01
.2	0.012005	0.016007	0.020008	0.024008	0.028008	0.032007	0.036006	0.040003	0.044000	0.047996	3996-02
.3	0.012006	0.016007	0.020009	0.024009	0.028009	0.032009	0.036007	0.040005	0.044002	0.047998	3996-01
.4	0.012006	0.016008	0.020010	0.024010	0.028011	0.032010	0.036009	0.040007	0.044004	0.048000	3996-02
.5	0.012007	0.016009	0.020011	0.024012	0.028012	0.032012	0.036011	0.040009	0.044006	0.048002	3996-02
.6	0.012008	0.016010	0.020012	0.024013	0.028013	0.032013	0.036013	0.040011	0.044008	0.048005	3997-02
.7	0.012008	0.016011	0.020013	0.024014	0.028015	0.032017	0.036015	0.040015	0.044013	0.048007	3997-03
.8	0.012009	0.016012	0.020014	0.024015	0.028016	0.032017	0.036017	0.040015	0.044013	0.048010	3997-03
.9	0.012010	0.016012	0.020015	0.024017	0.028018	0.032019	0.036019	0.040018	0.044016	0.048013	3997-02
3.0	0.012010	0.016013	0.020016	0.024018	0.028020	0.032021	0.036021	0.040020	0.044018	0.048016	3998-03
.1	0.012011	0.016014	0.020017	0.024020	0.028021	0.032023	0.036023	0.040022	0.044021	0.048018	3997-03
.2	0.012012	0.016015	0.020018	0.024021	0.028023	0.032025	0.036025	0.040025	0.044024	0.048022	3998-03
.3	0.012013	0.016016	0.020020	0.024023	0.028025	0.032027	0.036028	0.040028	0.044027	0.048025	3998-03
.4	0.012013	0.016017	0.020021	0.024024	0.028027	0.032029	0.036030	0.040030	0.044030	0.048028	3998-04

TABLE—Continued

y	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	Diff.
3.5	0.012014	0.016019	0.020023	0.024026	0.028029	0.032031	0.036032	0.040033	0.044033	0.048031	3998-05
.6	0.012015	0.016020	0.020024	0.024028	0.028031	0.032033	0.036035	0.040036	0.044036	0.048035	3999-05
.7	0.012016	0.016021	0.020025	0.024029	0.028032	0.032036	0.036038	0.040039	0.044039	0.048038	3999-05
.8	0.012017	0.016022	0.020027	0.024031	0.028035	0.032038	0.036040	0.040042	0.044042	0.048040	4000-05
.9	0.012018	0.016023	0.020028	0.024033	0.028037	0.032041	0.036043	0.040045	0.044046	0.048043	3999-05
4.0	0.012019	0.016025	0.020030	0.024035	0.028039	0.032043	0.036046	0.040048	0.044049	0.048049	4000-06
.1	0.012020	0.016026	0.020032	0.024037	0.028042	0.032046	0.036050	0.040051	0.044053	0.048053	4000-06
.2	0.012021	0.016027	0.020033	0.024039	0.028044	0.032048	0.036052	0.040055	0.044056	0.048057	4000-06
.3	0.012022	0.016029	0.020035	0.024041	0.028046	0.032051	0.036055	0.040058	0.044060	0.048061	4001-07
.4	0.012023	0.016030	0.020037	0.024043	0.028049	0.032054	0.036058	0.040062	0.044064	0.048065	4001-07
.5	0.012024	0.016031	0.020039	0.024045	0.028051	0.032057	0.036061	0.040065	0.044068	0.048070	4002-07
.6	0.012025	0.016033	0.020040	0.024047	0.028054	0.032060	0.036065	0.040069	0.044072	0.048074	4002-08
.7	0.012026	0.016034	0.020042	0.024050	0.028056	0.032063	0.036068	0.040072	0.044076	0.048079	4003-08
.8	0.012027	0.016036	0.020044	0.024052	0.028059	0.032066	0.036071	0.040076	0.044080	0.048083	4003-08
.9	0.012028	0.016037	0.020046	0.024054	0.028062	0.032069	0.036075	0.040080	0.044085	0.048088	4003-09
5.0	0.012030	0.016039	0.020048	0.024051	0.028065	0.032072	0.036079	0.040084	0.044089	0.048093	4004-09
.1	0.012031	0.016041	0.020050	0.024053	0.028068	0.032075	0.036082	0.040088	0.044093	0.048097	4004-10
.2	0.012032	0.016042	0.020052	0.024056	0.028070	0.032079	0.036086	0.040092	0.044098	0.048102	4004-10
.3	0.012033	0.016044	0.020054	0.024062	0.028073	0.032082	0.036090	0.040097	0.044103	0.048107	4004-11
.4	0.012035	0.016046	0.020056	0.024067	0.028076	0.032085	0.036094	0.040101	0.044107	0.048113	4006-11
.5	0.012036	0.016048	0.020059	0.024069	0.028079	0.032089	0.036097	0.040105	0.044112	0.048118	4006-12
.6	0.012037	0.016049	0.020061	0.024072	0.028083	0.032092	0.036102	0.040110	0.044117	0.048123	4006-12
.7	0.012039	0.016051	0.020063	0.024075	0.028086	0.032096	0.036106	0.040114	0.044122	0.048129	4007-12
.8	0.012040	0.016053	0.020065	0.024078	0.028089	0.032100	0.036110	0.040119	0.044127	0.048134	4007-13
.9	0.012041	0.016055	0.020068	0.024080	0.028092	0.032104	0.036114	0.040124	0.044132	0.048140	4008-14
6.0	0.012043	0.016057	0.020070	0.024083	0.028096	0.032107	0.036118	0.040128	0.044138	0.048146	4008-14
.1	0.012044	0.016059	0.020073	0.024086	0.028099	0.032111	0.036123	0.040133	0.044143	0.048151	4008-15
.2	0.012046	0.016061	0.020075	0.024090	0.028103	0.032115	0.036127	0.040138	0.044148	0.048157	4009-15
.3	0.012047	0.016063	0.020078	0.024092	0.028106	0.032119	0.036132	0.040143	0.044154	0.048163	4009-16
.4	0.012049	0.016065	0.020080	0.024095	0.028110	0.032123	0.036136	0.040148	0.044159	0.048170	4011-16
.5	0.012050	0.016067	0.020083	0.024098	0.028113	0.032128	0.036141	0.040154	0.044165	0.048176	4011-17
.6	0.012052	0.016069	0.020085	0.024102	0.028117	0.032132	0.036146	0.040159	0.044171	0.048182	4011-17
.7	0.012054	0.016071	0.020088	0.024105	0.028121	0.032136	0.036151	0.040164	0.044177	0.048189	4012-17
.8	0.012055	0.016073	0.020091	0.024108	0.028125	0.032140	0.036155	0.040170	0.044183	0.048195	4012-18
.9	0.012057	0.016076	0.020094	0.024111	0.028128	0.032145	0.036160	0.040175	0.044189	0.048202	4013-19

TABLE—Continued
 $\frac{1}{2}y-y'$

y	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	Diff.
7.0	0.012059	0.016078	0.020096	0.024115	0.028132	0.032149	0.036165	0.040181	0.044195	0.048208	4013-19
1	0.012060	0.016080	0.020099	0.024118	0.028136	0.032154	0.036171	0.040186	0.044201	0.048215	4014-20
2	0.012062	0.016082	0.020102	0.024122	0.028140	0.032158	0.036176	0.040191	0.044208	0.048222	4015-21
3	0.012064	0.016085	0.020105	0.024125	0.028144	0.032163	0.036181	0.040198	0.044214	0.048229	4016-22
4	0.012066	0.016087	0.020108	0.024129	0.028149	0.032168	0.036186	0.040204	0.044221	0.048236	4017-23
5	0.012067	0.016089	0.020111	0.024132	0.028153	0.032173	0.036192	0.040210	0.044227	0.048244	4018-24
6	0.012069	0.016092	0.020114	0.024136	0.028157	0.032178	0.036197	0.040216	0.044234	0.048251	4019-25
7	0.012070	0.016094	0.020117	0.024140	0.028161	0.032183	0.036203	0.040222	0.044241	0.048258	4020-26
8	0.012071	0.016097	0.020120	0.024143	0.028166	0.032188	0.036209	0.040229	0.044248	0.048266	4021-27
9	0.012075	0.016099	0.020124	0.024147	0.028170	0.032193	0.036214	0.040235	0.044255	0.048273	4022-28
8.0	0.012077	0.016102	0.020127	0.024151	0.028175	0.032198	0.036220	0.040241	0.044262	0.048281	4023-29
1	0.012079	0.016105	0.020130	0.024155	0.028179	0.032203	0.036226	0.040248	0.044269	0.048289	4024-30
2	0.012081	0.016107	0.020133	0.024159	0.028184	0.032208	0.036232	0.040255	0.044276	0.048297	4025-31
3	0.012083	0.016110	0.020137	0.024163	0.028189	0.032214	0.036238	0.040261	0.044284	0.048305	4026-32
4	0.012085	0.016113	0.020140	0.024167	0.028193	0.032219	0.036244	0.040268	0.044291	0.048313	4027-22
5	0.012087	0.016115	0.020143	0.024171	0.028198	0.032225	0.036250	0.040275	0.044299	0.048321	4028-23
6	0.012089	0.016118	0.020147	0.024175	0.028203	0.032230	0.036256	0.040282	0.044306	0.048330	4029-24
7	0.012091	0.016121	0.020151	0.024180	0.028208	0.032236	0.036263	0.040289	0.044314	0.048338	4030-25
8	0.012093	0.016124	0.020154	0.024184	0.028213	0.032241	0.036269	0.040296	0.044322	0.048347	4031-26
9	0.012095	0.016127	0.020158	0.024188	0.028218	0.032247	0.036276	0.040303	0.044330	0.048355	4032-27
9.0	0.012097	0.016130	0.020161	0.024192	0.028223	0.032253	0.036282	0.040310	0.044338	0.048364	4033-28
1	0.012100	0.016133	0.020165	0.024197	0.028228	0.032259	0.036289	0.040318	0.044346	0.048372	4034-29
2	0.012102	0.016135	0.020169	0.024201	0.028233	0.032265	0.036295	0.040325	0.044354	0.048382	4035-30
3	0.012104	0.016138	0.020172	0.024206	0.028239	0.032271	0.036302	0.040333	0.044362	0.048391	4036-31
4	0.012106	0.016142	0.020176	0.024210	0.028244	0.032277	0.036309	0.040340	0.044371	0.048400	4037-32
5	0.012109	0.016145	0.020180	0.024215	0.028249	0.032283	0.036316	0.040348	0.044379	0.048409	4038-33
6	0.012111	0.016148	0.020184	0.024220	0.028255	0.032289	0.036323	0.040356	0.044388	0.048418	4039-34
7	0.012114	0.016151	0.020188	0.024224	0.028260	0.032296	0.036330	0.040364	0.044396	0.048428	4040-35
8	0.012116	0.016154	0.020192	0.024229	0.028266	0.032302	0.036337	0.040372	0.044405	0.048437	4041-36
9	0.012118	0.016157	0.020196	0.024234	0.028272	0.032308	0.036344	0.040380	0.044414	0.048447	4042-37
10.0	0.012121	0.016161	0.020200	0.024239	0.028277	0.032315	0.036352	0.040388	0.044423	0.048457	4043-38

